BRADLEY'S MATHS

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(I)GCSE Extended Level Mathematics (0580)

E4.6 Angle Properties

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Abstract

Thank you for downloading this free taster resource from **Bradley's Maths**. I hope you and your students find it useful.

This worksheet is a sample from the comprehensive **E3 Coordinate Geometry & E4 Geometry** booklets, which together contain 20 worksheets covering every aspect of this section of the Cambridge IGCSE (0580) syllabus.

Each full booklet comes with a companion Answer Booklet containing fully worked, exam-style model answers and explanations for every question. Each worksheet and answer sheet has a Key Concepts and Formulas section with methods, pro-tips, galleries, deeper insights, cautionary notes, and in the answer sheets, reminders. These have been written with the student in mind in order to assist them in fully understanding the mathematics

All resources are meticulously crafted and professionally typeset using LaTeX for exceptional clarity and quality.

You can find the full E3 & E4 booklets and other resources by searching for "Bradley's Maths" on the TES website.

E3 COORDINATE GEOMETRY AND E4 GEOMETRY FREE TASTER RESOURCE

Instructions

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams, constructions, or graphs.
- Write your answer to each question in the space provided.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**. Give answers in degrees to **one decimal place**.
- You must state a clear geometrical reason for each step in your working.

Key Concepts: Angle Properties

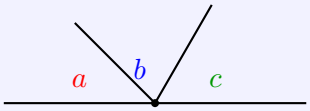
Deeper Insight: The Foundation of Geometry

Understanding the rules of how angles relate to each other is the absolute foundation of all geometry. These properties are not just facts to be memorised; they are logical certainties that allow you to solve complex problems and prove geometric truths. Mastering them is the first step to becoming a confident geometer.

Pro-Tip: The Three Basic Angle Rules

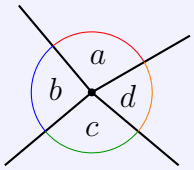
These three rules apply to any situation involving intersecting straight lines.

- **Angles on a Straight Line** add up to $180°$.



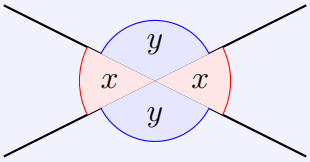
$$a + b + c = 180°$$

- **Angles at a Point** add up to $360°$.



$$a + b + c + d = 360°$$

- **Vertically Opposite Angles** are equal.

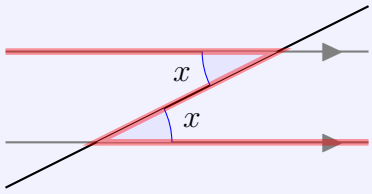


Vertically opposite angles are equal.
Also note: $x + y = 180°$ (Angles on a straight line)

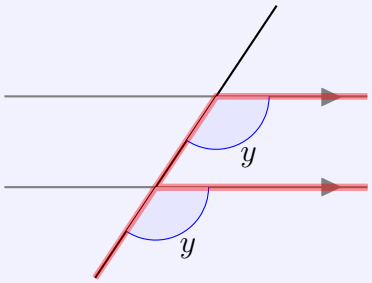
Pro-Tip: Angles in Parallel Lines (Z, F, and C Angles)

When a straight line (a **transversal**) cuts across two parallel lines, three powerful rules emerge. It is helpful to look for the letter shapes Z, F, and C to identify them.

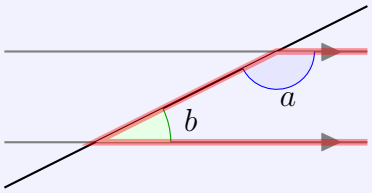
- **Alternate Interior Angles** are **EQUAL**. (Look for a 'Z' shape).



- **Corresponding Angles** are **EQUAL**. (Look for an 'F' shape).



- **Co-interior Angles ADD UP TO 180°**. (Look for a 'C' shape).



$$a + b = 180°$$

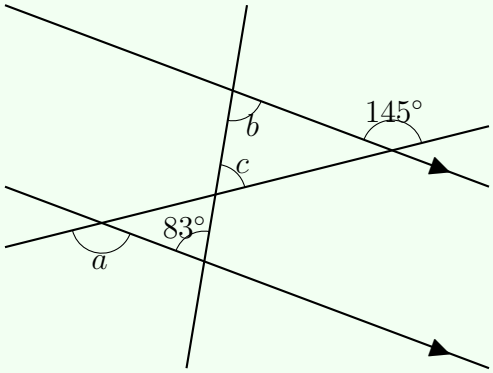
Caution: You MUST State Your Reasons!

In geometry questions, the answer alone is not enough. You will lose marks if you do not state the correct geometrical reason for each step of your working (e.g., "alternate angles are equal", "angles on a straight line add to 180°"). Be precise!

Example: Combining Angle Rules

The diagram shows two parallel lines intersecting two straight lines.

Find the values of a, b and c



Step 1: Find angle a.
The angle vertically opposite to the given $145°$ angle is also $145°$. This angle and angle a are corresponding angles (forming an 'F' shape).

$$a = \mathbf{145°}$$

Reason: *Corresponding angles are equal (via vertically opposite angles).*

Step 2: Find angle b.
Angle b and the given $83°$ angle are alternate interior angles (forming a 'Z' shape).

$$b = \mathbf{83°}$$

Reason: *Alternate interior angles are equal.*

Step 3: Find angle y.
The angles b, c, and the angle adjacent $145°$ on the straight line form a triangle in the centre of the diagram. First, find the third angle of this triangle (let's call it p).

$$p = 180° - 145° = 35°$$

Reason: *Angles on a straight line add up to* $180°$.
Now, the sum of the angles in the triangle is $180°$.

$$b + c + p = 180°$$

$$83° + c + 35° = 180°$$

$$c + 118° = 180°$$

$$c = 180° - 118° = \mathbf{62°}$$

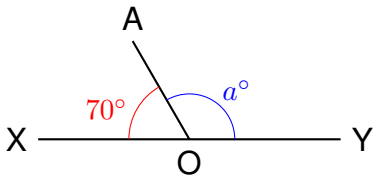
Reason: *The sum of angles in a triangle is* $180°$.

Step 4: Final Answers.
The calculated values for the angles are:

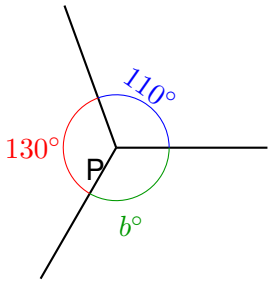
$$\boldsymbol{a = 145°}, \quad \boldsymbol{b = 83°}, \quad \boldsymbol{c = 62°}$$

1. For each of the diagrams below, calculate the value of the lettered angle(s). You **must** show your working and give a clear geometrical reason for each step. Lines that appear to be straight are indeed straight lines.

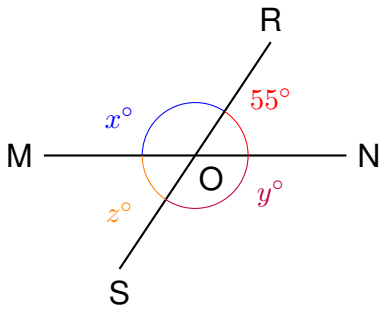
(a) [2]



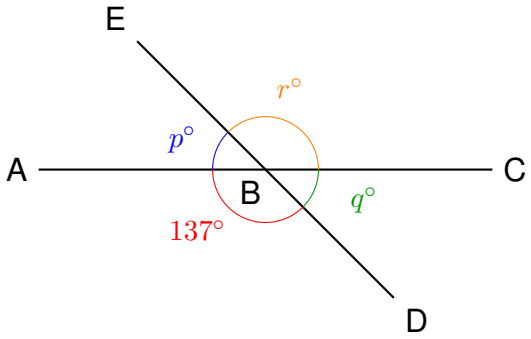
(b) [2]



(c) [4]



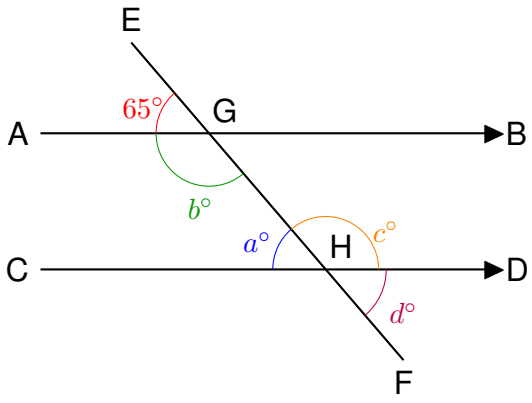
(d) [5]



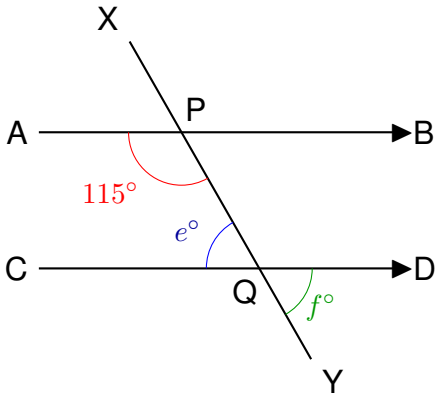
Total: [13]

2. In the diagrams below, lines AB and CD are parallel. Find the value of each lettered angle, giving a clear geometrical reason for each step.

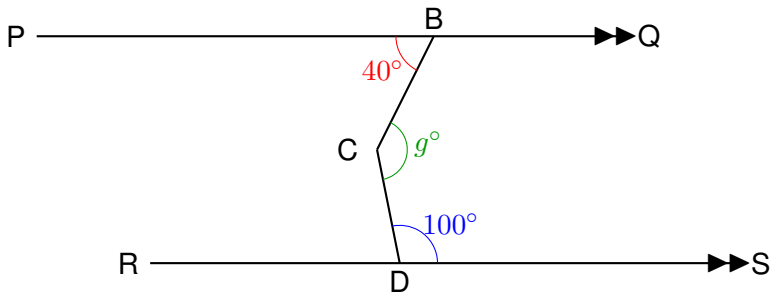
(a) Find the values of *a, b, c,* and *d*. [5]



(b) Find the values of *e* and *f*. [3]

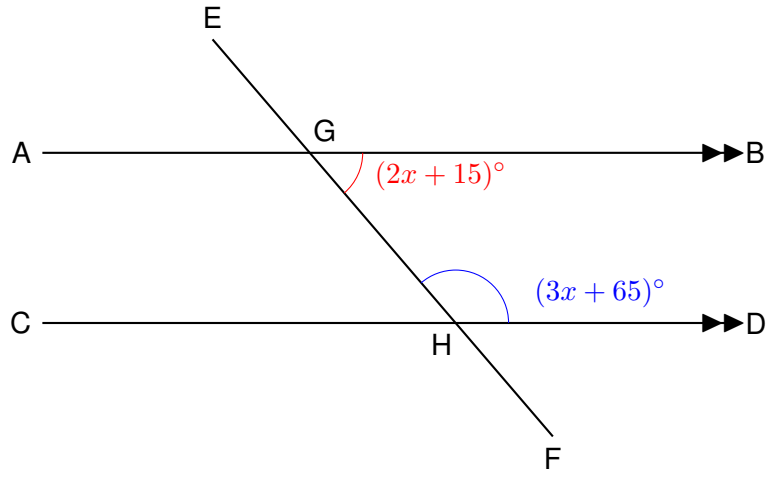


(c) Find the value of g. [3]



(Hint: Draw a line through C parallel to PQ and RS)

(d) Lines AB and CD are parallel.



(i) Form an equation in terms of x. [1]

(ii) Solve your equation to find the value of x. [2]

(iii) Find the size of $\angle GHD$. [1]

This is the end of the worksheet

1) Basic Angle Properties

Reminder: State Your Reasons!

In geometry, the answer is only half the solution. You must always state the correct geometrical reason for each step of your calculation to receive full marks.

(a)

$$a + 70^\circ = 180^\circ \qquad \text{(Angles on a straight line)}$$
$$a = 180^\circ - 70^\circ$$
$$a = \mathbf{110^\circ}$$

(b)

$$b + 130^\circ + 110^\circ = 360^\circ \qquad \text{(Angles at a point)}$$
$$b + 240^\circ = 360^\circ$$
$$b = 360^\circ - 240^\circ$$
$$b = \mathbf{120^\circ}$$

(c)

$$z = \mathbf{55^\circ} \qquad \text{(Vertically opposite angles)}$$
$$x + 55^\circ = 180^\circ \qquad \text{(Angles on a straight line)}$$
$$x = 180^\circ - 55^\circ = \mathbf{125^\circ}$$
$$y = x = \mathbf{125^\circ} \qquad \text{(Vertically opposite angles)}$$

(d)

$$p + 137^\circ = 180^\circ \qquad \text{(Angles on a straight line)}$$
$$p = 180^\circ - 137^\circ = \mathbf{43^\circ}$$
$$r = \mathbf{137^\circ} \qquad \text{(Vertically opposite angles)}$$
$$q = p = \mathbf{43^\circ} \qquad \text{(Vertically opposite angles)}$$

2) Angles in Parallel Lines

Pro-Tip: Choosing the Right Rule

When solving complex parallel line problems, there are often multiple correct paths. Look for the simplest rule first. Can you find a direct 'Z' (alternate) or 'F' (corresponding) angle relationship before you resort to multi-step solutions using straight lines or co-interior angles?

(a)

$a = \mathbf{65°}$	(Corresponding angles)
$b = 180° - 65° = \mathbf{115°}$	(Angles on a straight line)
$c = b = \mathbf{115°}$	(Alternate angles)
$d = a = \mathbf{65°}$	(Vertically opposite angles)

(b)

$e = 180° - 115° = \mathbf{65°}$	(Co-interior angles)
$f = 180° - e = 180° - 65° = \mathbf{115°}$	(Angles on a straight line)

(c)

Deeper Insight: The Power of Construction

The hint to draw an extra parallel line is a key problem-solving technique in geometry. By adding a line, you can break a complex, unfamiliar shape down into simpler shapes (in this case, two 'Z' angles) that you already know how to work with.

Draw a line through C parallel to PQ and RS. Let the part of angle g above the new line be g_1 and the part below be g_2.

$g_1 = 40°$	(Alternate angles)
Angle at D inside parallels $= 180° - 100° = 80°$	(Angles on a straight line)
$g_2 = 80°$	(Alternate angles)
$g = g_1 + g_2 = 40° + 80° = \mathbf{120°}$	

(d) (i) $(2x + 15) + (3x + 65) = 180$ (*Co-interior angles add to 180°*)

(ii)

$$\begin{aligned} 5x + 80 &= 180 \\ 5x &= 100 \\ x &= \mathbf{20} \end{aligned}$$

(iii) $\angle GHD = 3x + 65 = 3(20) + 65 = 125°$.