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(I)GCSE Extended Level Mathematics (0580)

E5.3b Arcs and Sectors

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Abstract

Thank you for downloading this free taster resource from **Bradley's Maths**. I hope you and your students find it useful.

This worksheet is a sample from the comprehensive **E5 Mensuration & E6 Trigonometry** booklets, which together contain 15 worksheets covering every aspect of this section of the Cambridge IGCSE (0580) syllabus.

Each full booklet comes with a companion Answer Booklet containing fully worked, exam-style model answers and explanations for every question. Each worksheet and answer sheet has a Key Concepts and Formulas section with methods, pro-tips, galleries, deeper insights, cautionary notes, and in the answer sheets, reminders. These have been written with the student in mind in order to assist them in fully understanding the mathematics

All resources are meticulously crafted and professionally typeset using LaTeX for exceptional clarity and quality.

You can find the full E5 & E6 booklets and other resources by searching for "Bradley's Maths" on the TES website.

E5 MENSURATION AND E6 TRIGONOMETRY FREE TASTER RESOURCE

Instructions

- Answer all questions in the spaces provided.
- For every calculation, you must write down the formula you are using first.
- Use the π button on your calculator unless the question specifies otherwise.
- Give final answers correct to 3 significant figures, unless specified a different degree of accuracy.
- Diagrams are not drawn to scale.

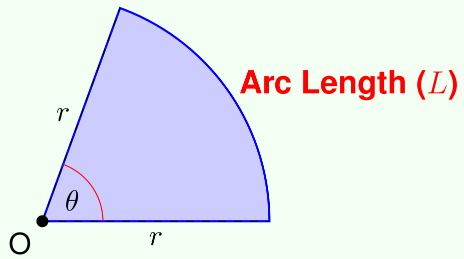
Key Concepts: Arcs and Sectors

Deeper Insight: Slices of a Circle

An arc is a piece of a circle's circumference, and a sector is a slice of its area. Both are defined by an angle (θ) at the centre. The key to all calculations is to think of them as a **fraction of a full circle**. The fraction is always the angle of the slice out of the full 360 degrees: $\frac{\theta}{360}$.

Method: The Core Formulas

To find the arc length or area of a sector, simply multiply the "fraction of the circle" by the formula for the full circle.



a) Arc Length (L)
(A fraction of the Circumference)

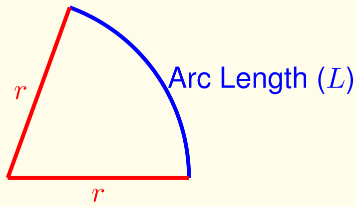
$$L = \frac{\theta}{360} \times 2\pi r$$

b) Area of a Sector (A)
(A fraction of the Area)

$$A = \frac{\theta}{360} \times \pi r^2$$

Caution: The Perimeter of a Sector Trap

The perimeter is the total distance **around** the shape. For a sector, this is NOT just the arc length. You must remember to add the two straight radii.



$$\text{Perimeter} = (\text{Arc Length}) + (\text{Radius}) + (\text{Radius}) = \boldsymbol{L + 2r}$$

Problem-Solving with Arcs and Sectors

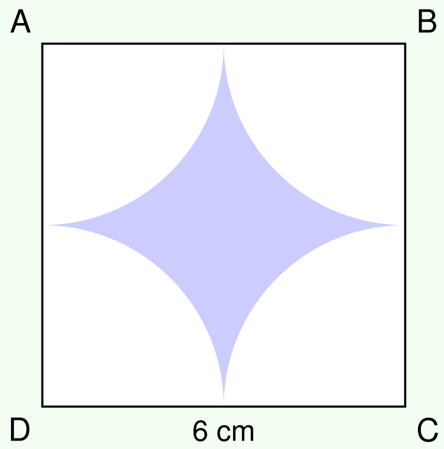
Pro-Tip: Working Backwards

If you are given the arc length or area, you will need to rearrange the formulas to find the radius (r) or the angle (θ). For example, to find θ from the arc length L:

$$L = \frac{\theta}{360} \times 2\pi r \quad \Longrightarrow \quad \boldsymbol{\theta = \frac{L \times 360}{2\pi r}}$$

Example: Finding a Shaded Area

Problem: The diagram shows a square ABCD with side length 6 cm. Four identical quarter-circles are removed from each corner. Calculate the area of the shaded region. Leave your answer in terms of π.



Step 1: The Strategy: Total Area - Unshaded Area.
The shaded area is the Area of the Square minus the area of the four unshaded quarter-circles.

Step 2: Combine the Unshaded Parts.
The four unshaded quarter-circles are identical. Together, four quarter-circles make one full circle. The radius of each quarter-circle is half the side length of the square.

$$\text{Radius } r = 6 \div 2 = 3 \text{ cm}$$

Step 3: Calculate the Areas.

$$\text{Area}_{\text{Square}} = 6 \times 6 = 36 \text{ cm}^2$$
$$\text{Area}_{\text{Full Circle}} = \pi r^2 = \pi \times 3^2 = 9\pi \text{ cm}^2$$

Step 4: Calculate the Shaded Area.

$$\text{Shaded Area} = 36 - 9\pi$$

Answer: $(36 - 9\pi)$ **cm²**

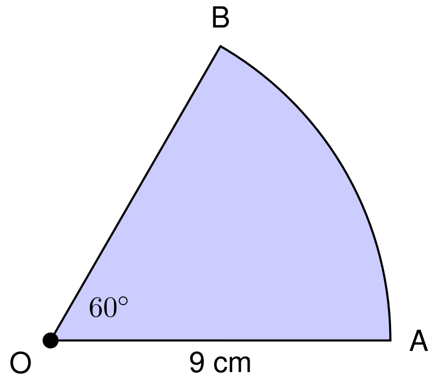
1. For each sector shown below, calculate:

   (i) the length of the arc

   (ii) the area of the sector.

   Give your answers in terms of π.

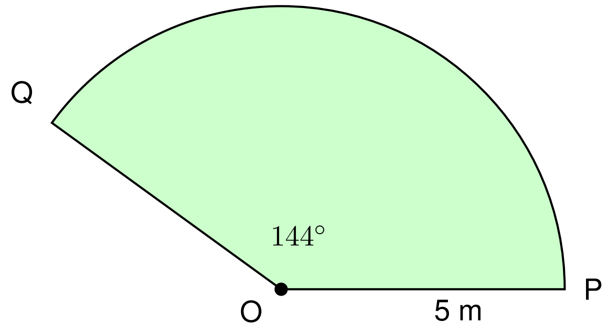
   (a) Sector AOB with radius 9 cm and angle AOB = $60°$.



   (i) Arc length: [2]

   (ii) Area of sector: [2]

(b) Sector POQ with radius 5 m and angle POQ = $144°$.



(i) Arc length: [2]

(ii) Area of sector: [2]

Total: [8]

2. A sector of a circle has radius 12 cm and arc length 10π cm.

   (a) Calculate the angle θ of the sector. [2]

   (b) Calculate the area of this sector. Give your answer in terms of π. [2]

Total: [4]

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3. The area of a sector of a circle is 24π cm^2. The angle of the sector is $120°$. Calculate the radius of the circle. [3]

Total: [3]

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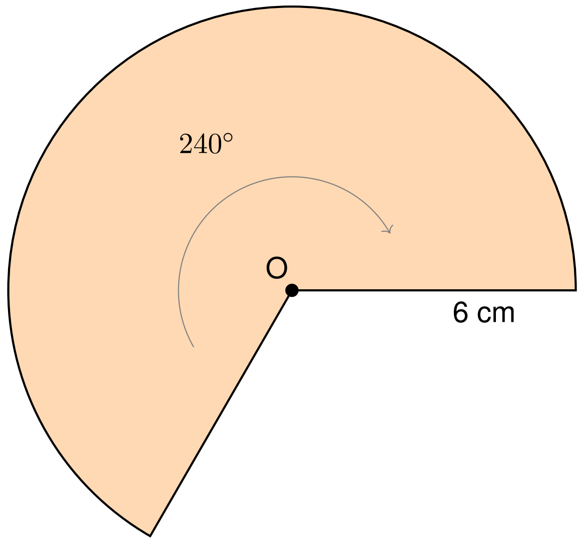
4. The diagram shows a sector of a circle with centre O and radius 6 cm. The angle of the sector is $240°$.



(a) Is this a minor or a major sector? Explain your answer. [1]

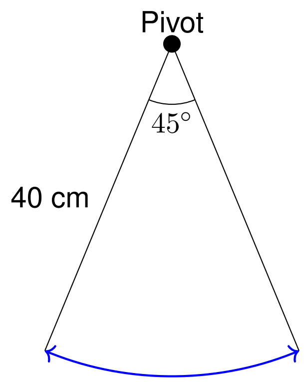
(b) Calculate the length of the major arc. (Use the π button on your calculator) [2]

(c) Calculate the area of this sector. (Use the π button on your calculator) [2]

(d) Calculate the perimeter of this sector. (Use the π button on your calculator) [2]

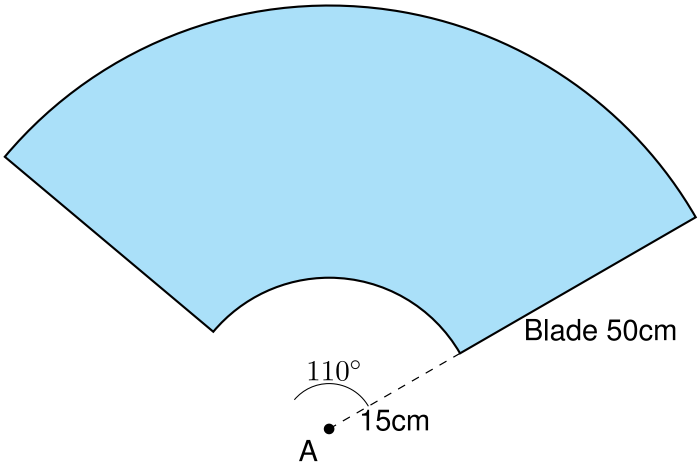
Total: [7]

5. A pendulum swings through an angle of $45°$. The length of the pendulum is 40 cm. Calculate the length of the arc the end of the pendulum swings through. Give your answer correct to 1 decimal place. (Use the π button on your calculator) [2]



Total: [2]

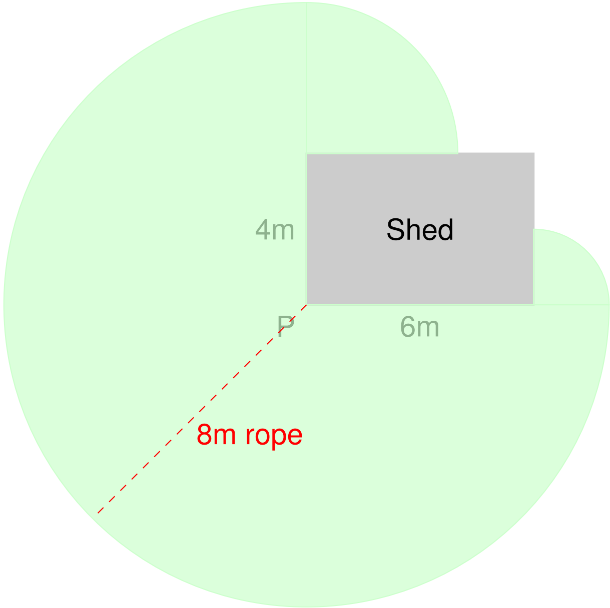
6. The diagram shows a windscreen wiper on a car. The wiper blade is 50 cm long and it sweeps through an angle of $110°$. The blade itself starts 15 cm from the pivot point A.



Calculate the area of the windscreen cleared by the wiper blade. Give your answer correct to 3 significant figures. (Use the π button on your calculator) [4]

Total: [4]

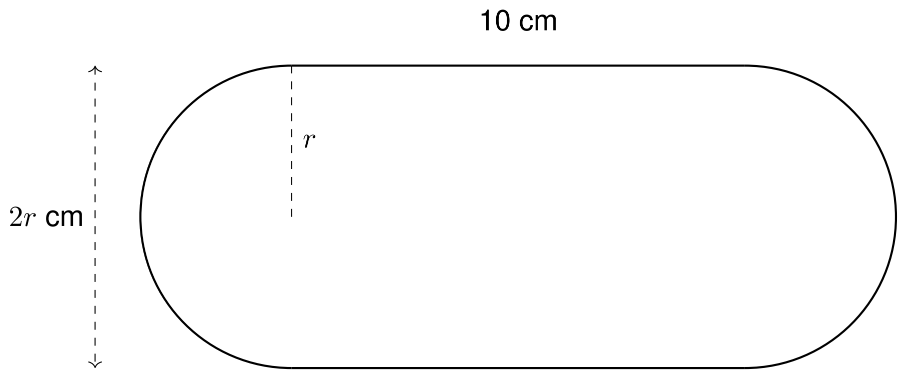
7. A goat is tethered to a corner, P, of a rectangular shed measuring 6 m by 4 m. The rope is 8 m long. The shed is in the middle of a large field of grass.



Calculate the total area in which the goat can graze. Give your answer correct to 3 significant figures. (Use the π button on your calculator) [5]

Total: [5]

8. The diagram shows a shape made from a rectangle and two identical semicircles. The rectangle has length 10 cm and width $2r$ cm. Each semicircle has radius r cm. The perimeter of the whole shape is P cm.

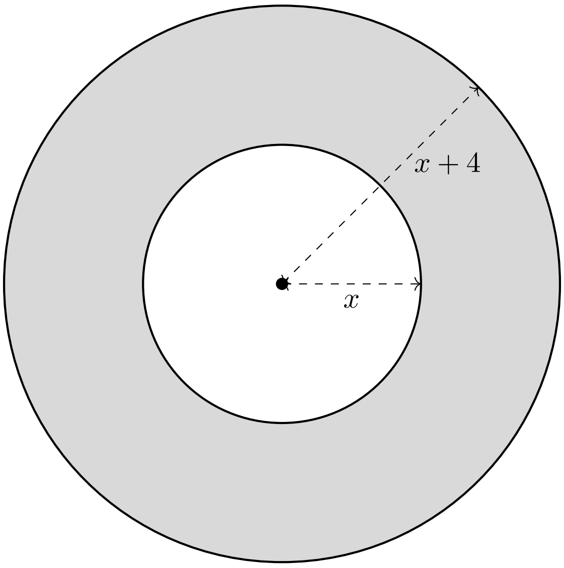


(a) Find an expression for P in terms of r and π. Simplify your answer. [3]

(b) The perimeter P is 50 cm. Calculate the value of r. (Use the π button on your calculator) [2]

Total: [5]

9. A circular disc is removed from the centre of a larger circular piece of metal. The radius of the smaller disc is x cm. The radius of the larger piece of metal is $(x+4)$ cm. The area of metal remaining (the annulus) is 40π cm^2.



Find the value of x. [4]

Total: [4]

This is the end of the worksheet

Worked Solutions

1)

Reminder: Core Formulas

Remember the two key formulas:

- Arc Length: $L = \frac{\theta}{360} \times 2\pi r$
- Sector Area: $A = \frac{\theta}{360} \times \pi r^2$

(a) Sector AOB with $r = 9$ cm and $\theta = 60°$.

(i) **Arc Length:**

$$L = \frac{60}{360} \times 2\pi \times 9 = \frac{1}{6} \times 18\pi = \mathbf{3\pi \text{ cm}}$$

(ii) **Area of Sector:**

$$A = \frac{60}{360} \times \pi \times 9^2 = \frac{1}{6} \times 81\pi = \mathbf{13.5\pi \text{ cm}^2}$$

(b) Sector POQ with $r = 5$ m and $\theta = 144°$.

(i) **Arc Length:**

$$L = \frac{144}{360} \times 2\pi \times 5 = \frac{2}{5} \times 10\pi = \mathbf{4\pi \text{ m}}$$

(ii) **Area of Sector:**

$$A = \frac{144}{360} \times \pi \times 5^2 = \frac{2}{5} \times 25\pi = \mathbf{10\pi \text{ m}^2}$$

2)

(a) Find the angle θ. We are given $r = 12$ and $L = 10\pi$.

Pro-Tip: Working Backwards

Rearrange the arc length formula to make θ the subject: $\theta = \frac{L \times 360}{2\pi r}$.

$$10\pi = \frac{\theta}{360} \times 2\pi \times 12$$
$$10\pi = \frac{\theta}{360} \times 24\pi \qquad \text{(Simplify)}$$
$$\frac{10}{24} = \frac{\theta}{360} \qquad \text{(Divide both sides by } 24\pi\text{)}$$
$$\theta = \frac{10}{24} \times 360 = \mathbf{150^\circ}$$

(b) Find the area. Now use $\theta = 150^\circ$ and $r = 12$.

$$A = \frac{150}{360} \times \pi \times 12^2 = \frac{5}{12} \times 144\pi = \mathbf{60\pi\ cm^2}$$

3) We are given $A = 24\pi$ and $\theta = 120^\circ$. Rearrange the area formula to find r.

$$24\pi = \frac{120}{360} \times \pi r^2$$
$$24\pi = \frac{1}{3}\pi r^2 \qquad \text{(Simplify the fraction and cancel } \pi\text{)}$$
$$24 = \frac{1}{3}r^2 \qquad \text{(Multiply both sides by 3)}$$
$$72 = r^2$$
$$r = \sqrt{72} = \sqrt{36 \times 2} = \mathbf{6\sqrt{2}\ cm}$$

4)

(a) A major sector has an angle greater than $180°$. A minor sector has an angle less than $180°$. As $240° > 180°$, this is a **major sector**.

(b) **Arc Length:** $(r = 6, \theta = 240)$

$$L = \frac{240}{360} \times 2\pi \times 6 = \frac{2}{3} \times 12\pi = 8\pi = 25.132... \approx \mathbf{25.1\ cm}\ \text{(3 s.f.)}$$

(c) **Area of Sector:**

$$A = \frac{240}{360} \times \pi \times 6^2 = \frac{2}{3} \times 36\pi = 24\pi = 75.398... \approx \mathbf{75.4\ cm^2}\ \text{(3 s.f.)}$$

(d) **Perimeter:**

> Caution: Don't Forget the Radii!
>
> The perimeter of a sector is the arc length PLUS the two straight radii. $P = L + 2r$.

Use the unrounded value for the arc length, 8π.

$$P = 8\pi + (2 \times 6)$$
$$P = 25.132... + 12 = 37.132...$$
$$\approx \mathbf{37.1\ cm} \quad \text{(to 3 s.f.)}$$

5) This is an arc length problem with $r = 40$ cm and $\theta = 45°$.

$$L = \frac{45}{360} \times 2\pi \times 40$$
$$L = \frac{1}{8} \times 80\pi = 10\pi$$
$$L = 31.415... \approx \mathbf{31.4\ cm} \quad \text{(to 1 d.p.)}$$

6)

Pro-Tip: Area of a 'Wiper' Shape

The strategy is always: Area of Large Sector - Area of Small Sector. You can factorise the formula to make the calculation quicker.

Large sector radius, $R = 15 + 50 = 65$ cm.

Small sector radius, $r = 15$ cm.

Angle for both, $\theta = 110°$.

$$\text{Area} = \left(\frac{110}{360} \times \pi R^2\right) - \left(\frac{110}{360} \times \pi r^2\right)$$

$$= \frac{110}{360}\pi(R^2 - r^2) \qquad \text{(Factorise)}$$

$$= \frac{11}{36}\pi(65^2 - 15^2)$$

$$= \frac{11}{36}\pi(4225 - 225)$$

$$= \frac{11}{36}\pi(4000) = \frac{11000\pi}{9}$$

$$= 3839.72... \approx \mathbf{3840\ cm^2} \quad \text{(to 3 s.f.)}$$

7)

Pro-Tip: Deconstructing the Grazing Area

Break the total area into the sum of the simple shapes the goat can reach. The rope will bend around the corners of the shed.

The area is the sum of three sectors:

- **Sector 1:** A large $\frac{3}{4}$ circle with radius $r_1 = 8$ m. Angle $\theta_1 = 270°$.
- **Sector 2:** A quarter-circle with radius $r_2 = 8 - 6 = 2$ m. Angle $\theta_2 = 90°$.
- **Sector 3:** A quarter-circle with radius $r_3 = 8 - 4 = 4$ m. Angle $\theta_3 = 90°$.

$$\text{Total Area} = \left(\frac{270}{360}\pi \times 8^2\right) + \left(\frac{90}{360}\pi \times 2^2\right) + \left(\frac{90}{360}\pi \times 4^2\right)$$

$$= \left(\frac{3}{4} \times 64\pi\right) + \left(\frac{1}{4} \times 4\pi\right) + \left(\frac{1}{4} \times 16\pi\right)$$

$$= 48\pi + \pi + 4\pi = 53\pi$$

$$= 166.50... \approx \mathbf{167\ m^2} \quad \text{(to 3 s.f.)}$$

8)

(a) Perimeter: This consists of two straight sides (10 cm each) and two semi-circles, which make one full circle with radius r.

$$P = (2 \times 10) + (2\pi r)$$
$$P = \mathbf{20 + 2\pi r}$$

(b) Find r when $P = 50$:

$$50 = 20 + 2\pi r$$
$$30 = 2\pi r \qquad \text{(Subtract 20 from both sides)}$$
$$r = \frac{30}{2\pi} = \frac{15}{\pi}$$
$$r = 4.774... \approx \mathbf{4.77\ cm} \quad \text{(to 3 s.f.)}$$

9)

> **Pro-Tip: Difference of Two Squares**
>
> The area of an annulus is $A = \pi R^2 - \pi r^2$, which can be factorised to $A = \pi(R^2 - r^2)$. This is often quicker to calculate.

Large radius $R = x + 4$. Small radius $r = x$. Area = 40π.

$$40\pi = \pi(x+4)^2 - \pi x^2 \qquad \text{(Divide both sides by } \pi\text{)}$$
$$40 = (x+4)^2 - x^2$$
$$40 = (x^2 + 8x + 16) - x^2 \qquad \text{(Expand the bracket)}$$
$$40 = 8x + 16$$
$$24 = 8x$$
$$x = \mathbf{3}$$