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(I)GCSE Extended Level Mathematics (0580)

E2.10b Solving Equations Graphically

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Abstract

Thank you for downloading this free taster resource from **Bradley's Maths**. I hope you and your students find it useful.

This worksheet is a sample from the comprehensive **E2 Algebra and Graphs** booklet, which contains 32 worksheets covering every aspect of this section of the Cambridge IGCSE (0580) syllabus.

Each full booklet comes with a companion Answer Booklet containing fully worked, exam-style model answers and explanations for every question. Each worksheet and answer sheet has a Key Concepts and Formulas section with methods, pro-tips, galleries, deeper insights, cautionary notes, and in the answer sheets, reminders. These have been written with the student in mind in order to assist them in fully understanding the mathematics

All resources are meticulously crafted and professionally typeset using LaTeX for exceptional clarity and quality.

You can find the full E2 Algebra and Graphs booklet and other resources by searching for "Bradley's Maths" on the TES website.

E2 ALGEBRA AND GRAPHS FREE TASTER RESOURCE

Instructions

- Answer all questions.
- Show all your working clearly in the spaces provided.
- The number of marks is given in brackets [ ] at the end of each question or part question.
- A calculator **is permitted** for calculating table values if needed, but graphical work should be done accurately by hand.
- Use pencil for drawing graphs and lines. Use a ruler for straight lines.
- Use the graphs provided to solve the equations. Clearly draw any additional lines used on the graph.
- Read values from the graphs as accurately as possible.

Key Concepts: Solving Equations Using Graphs

Deeper Insight: The Power of a Graph

A drawn graph of an equation, like $y = f(x)$, is a powerful visual tool. It is a complete picture of every possible solution to that equation. We can use this picture to solve other, more complex equations by finding where new lines or curves intersect with our original graph.

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1. Solving Equations of the Form $f(x) = k$

Method: Solving $f(x) =$ a number

This is the most straightforward case, where we find the x-values for which our function equals a constant number, k.

1. On the same axes as your curve $y = f(x)$, draw the **horizontal line $y = k$**.
2. Find the points where this new line intersects the original curve.
3. The ***x*-coordinates** of these intersection points are the solutions.

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2. Solving More Complex Equations by Rearrangement

Method: Solving by Algebraic Rearrangement

The key is to algebraically manipulate a new equation until one side is **identical** to the expression for the curve you have already drawn.

1. Start with the new equation you have been asked to solve.
2. Rearrange it by adding or subtracting terms from **both sides** until one side exactly matches the expression for your curve, $f(x)$.
3. The other side of the equation will now be a new function, let's call it $g(x)$. You now have an equation of the form $\boldsymbol{f(x) = g(x)}$.
4. The solutions are the x-coordinates of the points where the graph of $y = f(x)$ intersects the graph of $\boldsymbol{y = g(x)}$.
5. Draw the graph of $y = g(x)$ (which is usually a straight line) on the same axes and read off the x-coordinates of the intersection points.

Example: Using the graph of $y = x^2 - 2x - 3$, solve the equation $x^2 - 3x - 1 = 0$.

Our goal is to rearrange $x^2 - 3x - 1 = 0$ so that the left-hand side becomes $x^2 - 2x - 3$. We compare the terms:

- To change $-3x$ into $-2x$, we must add x.
- To change -1 into -3, we must subtract 2.

Therefore, we must add $(x-2)$ to **both sides** of the equation:

$$x^2 - 3x - 1 = 0$$
$$(x^2 - 3x - 1) + (x - 2) = 0 + (x - 2)$$
$$x^2 - 2x - 3 = x - 2$$

The problem is now to find where the original curve $y = x^2 - 2x - 3$ intersects the new line $\boldsymbol{y = x - 2}$. We would draw this line on the graph to find the solutions.

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3. Exponential Graphs ($\boldsymbol{y = a^x}$)

Pattern Spotting: The Exponential Curve

Exponential functions produce a distinct curve with key features you should learn to recognise:

- The curve is always above the x-axis (the y-values are always positive).
- It always passes through the point $(0, 1)$, because any number to the power of 0 is 1 (e.g., $a^0 = 1$).
- The curve gets very steep very quickly as x increases (exponential growth).

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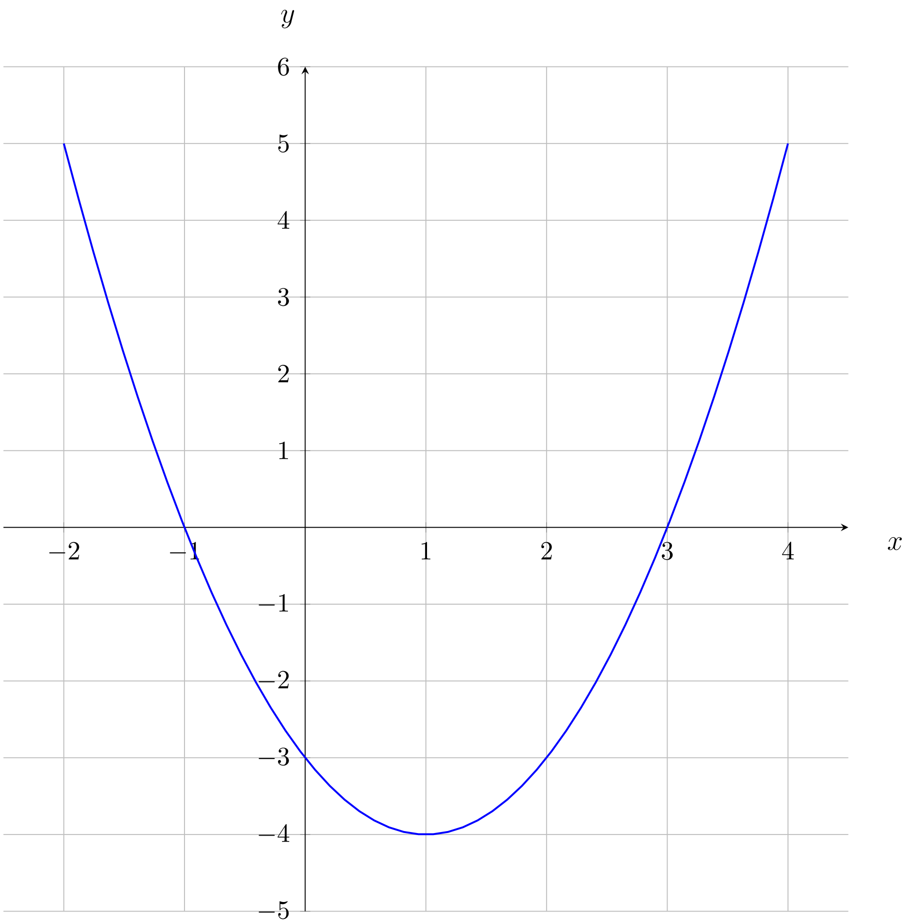
1. The graph of $y = x^2 - 2x - 3$ for $-2 \le x \le 4$ is shown below.



Use the graph to solve the following equations:

(a) $x^2 - 2x - 3 = 2$ [1]

(b) $x^2 - 2x - 3 = -4$ [1]

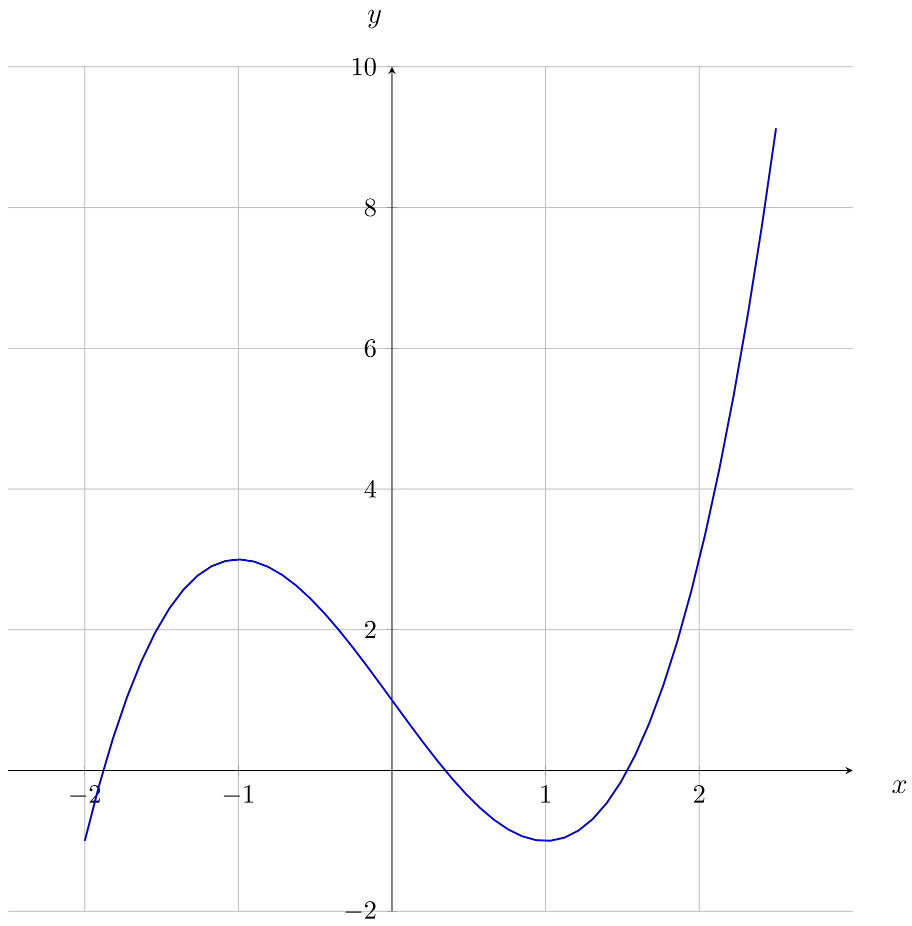
(c) $x^2 - 2x = 0$ [2]

(d) $x^2 - 3x - 1 = 0$ (Hint: Rearrange to $x^2 - 2x - 3 = x - 2$) [3]

(e) $x^2 - 4 = 0$ (hint: Start with $x^2 - 4 = 0$ and rearrange it to get $x^2 - 2x - 3 =$ something. Draw the graph of the ”something” and write down the solutions. Note, the available marks are for the algebraic working and the graph, not for the solutions! [4]

Total: [7]

2. The graph of $y = x^3 - 3x + 1$ for $-2 \leq x \leq 2.5$ is shown below.



Use the graph to find the approximate solutions to the following equations:

(a) $x^3 - 3x + 1 = 4$ [1]

(b) $x^3 - 3x - 1 = 0$ [2]

(c) $x^3 - 4x + 1 = 0$ [3]

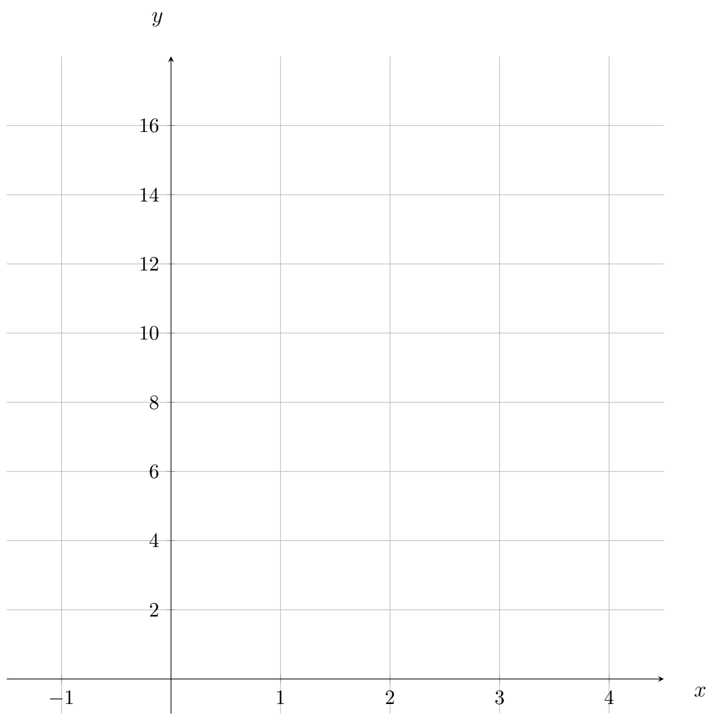
Total: [6]

3. Consider the equation $y = 2^x$.

(a) Complete the table of values. [2]

x	-1	0	1	2	3	3.5	4
y	0.5		2		8		

(b) On the grid below, draw the graph of $y = 2^x$ for $-1 \leq x \leq 4$. [3]



(c) Use your graph to estimate the value of $2^{2.5}$. [1]

(d) Use your graph to find the approximate value of x when $y = 10$. [1]

Total: [7]

This is the end of the worksheet

Model Answers

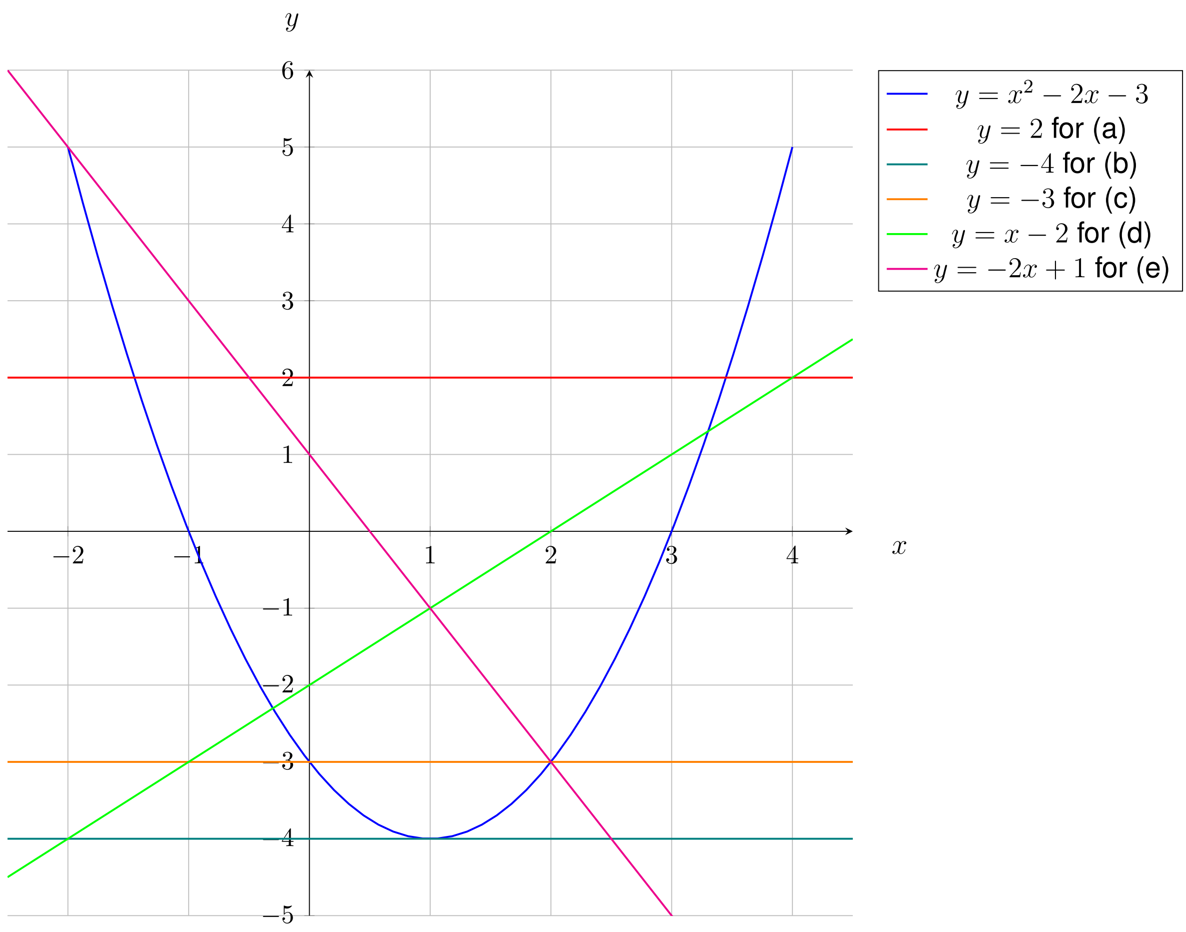
1)

Reminder: The Core Principle

To solve an equation using a given graph, you must rearrange the new equation so that one side is identical to the expression for the graph you have been given.

The other side of the equation forms a **new line** that you must draw. The solutions are the *x-coordinates* of the points where the original curve and your new line intersect.

The graph below shows the original curve $y = x^2 - 2x - 3$ in blue, along with the lines required to solve each part of the question.



(a) To solve $x^2 - 2x - 3 = 2$, we draw the line $y = 2$ (shown in red). The solutions are the x-coordinates of the intersection points.

Answer: $\boldsymbol{x \approx -1.4}$ **and** $\boldsymbol{x \approx 3.4}$.

(b) To solve $x^2 - 2x - 3 = -4$, we draw the line $y = -4$ (shown in teal). This line touches the graph at its minimum point.

Answer: $\boldsymbol{x = 1}$.

(c) Rearrange the equation: $x^2 - 2x = 0 \implies x^2 - 2x - 3 = -3$. We draw the line $y = -3$ (shown in orange).

Answer: $\boldsymbol{x = 0}$ and $\boldsymbol{x = 2}$.

(d) Rearrange the equation: $x^2 - 3x - 1 = 0 \implies x^2 - 2x - 3 = x - 2$. We draw the line $y = x - 2$ (shown in green).

Answer: $\boldsymbol{x \approx -0.5}$ and $\boldsymbol{x \approx 3.5}$.

(e)

Caution: Careful Algebraic Manipulation is Key

The marks are often for the algebraic working, not just the final answer. You must show how you rearrange the equation to match the original graph.

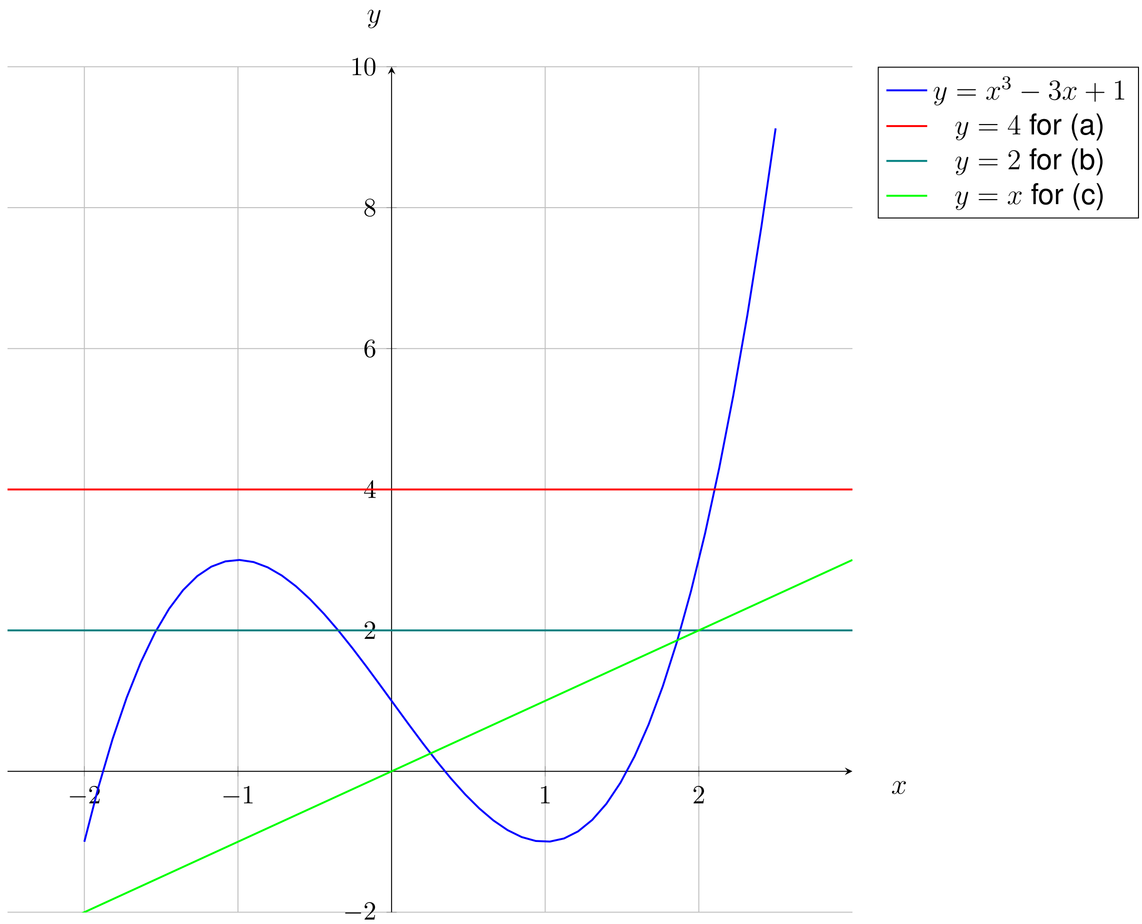
We need to make the left side of $x^2 - 4 = 0$ look like $x^2 - 2x - 3$. To do this, we can add and subtract terms, but we must do the same to both sides to keep the equation balanced. A methodical way is to add the terms we need, and then subtract them again.

$$\begin{aligned} x^2 - 4 &= 0 \\ (x^2 - 2x - 3) + 2x + 3 - 4 &= 0 \\ x^2 - 2x - 3 + 2x - 1 &= 0 \\ x^2 - 2x - 3 &= -2x + 1 \end{aligned}$$

We must draw the line $y = -2x + 1$ (shown in magenta).

Answer: The intersection points are at $\boldsymbol{x = -2}$ and $\boldsymbol{x = 2}$.

2) The graph below shows the original curve $y = x^3 - 3x + 1$ in blue, along with the lines required to solve each part.



(a) To solve $x^3 - 3x + 1 = 4$, we draw the line $y = 4$ (red).

Answer: $\boldsymbol{x \approx 2.2}$.

(b) Rearrange: $x^3 - 3x - 1 = 0 \implies x^3 - 3x + 1 = 2$. Draw the line $y = 2$ (teal).

Answer: $\boldsymbol{x \approx -1.5}$, $\boldsymbol{x \approx -0.7}$, and $\boldsymbol{x \approx 2.1}$.

(c) Rearrange: $x^3 - 4x + 1 = 0 \implies x^3 - 3x + 1 = x$. Draw the line $y = x$ (green).

Answer: $\boldsymbol{x \approx -2.1}$, $\boldsymbol{x \approx 0.25}$, and $\boldsymbol{x \approx 1.9}$.

3)

Pro-Tip: Using a Graph as a Calculator

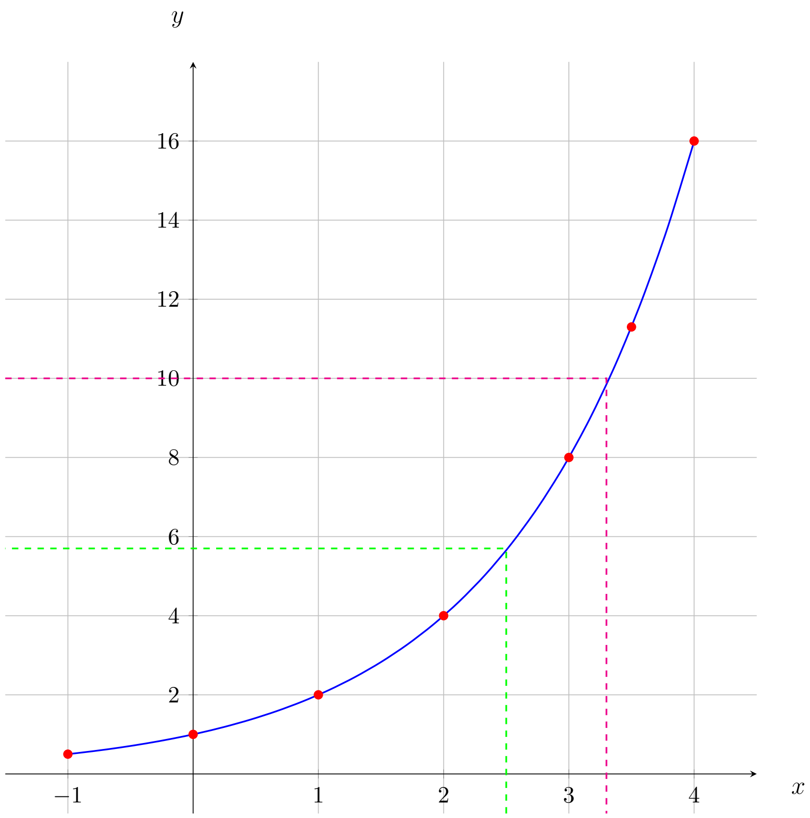
Once a graph like $y = 2^x$ is drawn, it becomes a powerful tool. You can use it to estimate values that are not in your table, both for powers (reading from the x-axis to the y-axis) and for roots/logarithms (reading from the y-axis to the x-axis).

(a) To complete the table for $y = 2^x$:

- When $x = 0, y = 2^0 = 1$.
- When $x = 2, y = 2^2 = 4$.
- When $x = 3.5, y = 2^{3.5} \approx 11.3$.
- When $x = 4, y = 2^4 = 16$.

x	-1	0	1	2	3	3.5	4
y	0.5	**1**	2	**4**	8	**11.3**	**16**

(b) The graph of $y = 2^x$ is plotted below, with points from the table marked in red.



(c) To find $2^{2.5}$, we find $x = 2.5$ and read the y-value from the graph (shown in green).

Answer: $\boldsymbol{y \approx 5.7}$.

(d) To find x when $y = 10$, we find $y = 10$ and read the x-value from the graph (shown in magenta).

Answer: $\boldsymbol{x \approx 3.3}$.